

# The Pi Waltz

There are a few ways to quickly dive in to the melodic structure to see its most interesting aspects.

#1. Play each line (8 measure phrase) independently, slowly, noticing how frequently the melody notes drive the chords.

You often only need one note of harmony to support the melody. Lead in to each phrase from the exit of the previous phrase.

#2. Play the chord progression by itself. Notice the common tones as you try to voice lead it.

$E\flat$     $A\flat$     $Cm$     $G7$     $A\flat$     $B\flat$     $Fm7$     $B\flat7$



The first 17 digits, with the exception of the B natural, hold to an ionian scale based in "4" or  $E\flat$ .

The melodic phrase was revealed simply by playing the digits as quarter notes.

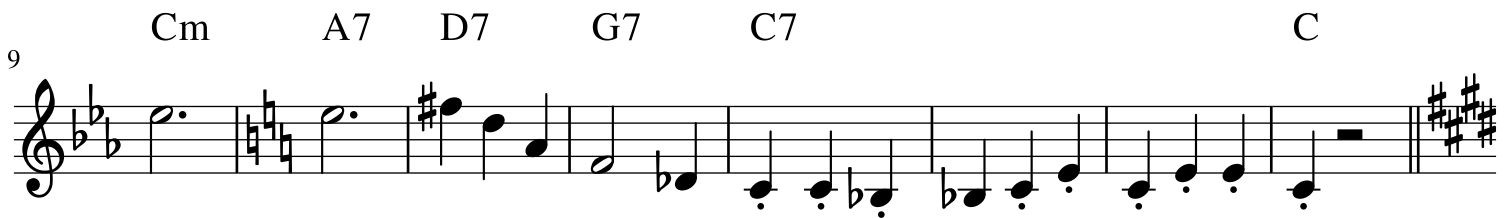
Measures 5-7 have groups of 3 notes with melodic direction on their own that make the end of a phrase.

The first chord was chosen because that's the tonal center of these 16 digits.

I IV vi III IV V ii V ... a progression (that stands on its own) emerges.

The melody notes in measures #1 and #2 are echoed at the beginning of measures #6 and #7. Simliar interval jumps from measure #3 are echoed in measure #5. There is a walk-down in measures #6 and #7.

Take a moment to consider the notes a 12 sided die would produce, next to this.



The melody is making a shift towards C, so I made that shift to  $Cm$  to open the section.

From there, it goes through ... i VI II V I

The  $A7$  starts a descending tritone ( $G + C\#$ )

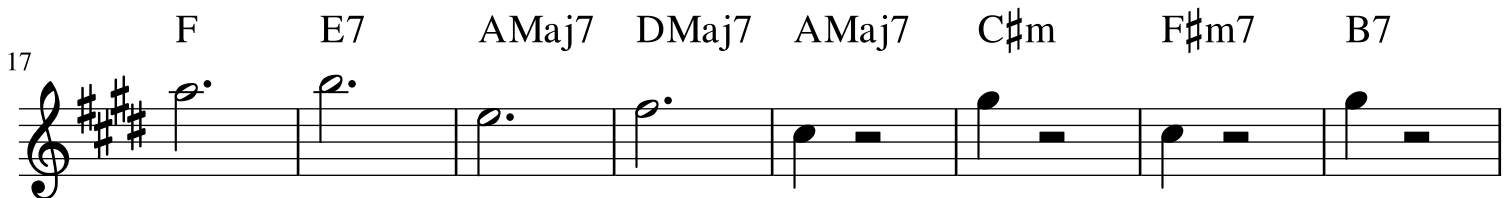
The  $D7$  has the next tritone ( $F\# + C$ )

The  $G7$  has the next one ( $F + B$ )

The melody hits a dissonant note ( $D\flat$ ) before snatching up the final tritone ( $B\flat + E$ )

33 digits in. 2 phrases with some very interesting structure.

Take a moment to consider what 33 rolls of a 12 sided die would produce at this point.



From  $C7$  to  $F$  to open the phrase. This phrase and the next are centered in the key of E major.

It's tricky to say exactly where key changes happen. The goal is to minimize accidentals in that regard, or to use the most useful key signatures.

This is the only 8 note phrase.

25

E      F#m      B7      E      B7      F#7      G#7(no3) C#7

Coming from a B7, this phrase opens in E, and uses a lovely accidental in the 3rd measure ... I ii V I ...  
 From there, the melody notes drive the harmony down in fourths (E B7 F#7) until subtly shifting tonal centers.

There is the start of an echo between this phrase and the next. The last note of this phrase is a B and the first note of the next phrase is a hard A right below it...

33

F#m      C#7      D      A      Dm      G      Dm      G

C#7 in the previous section opening to F#m. The 2nd measure ends with a B, and moves to a hard A. That is the answer echo from the previous section.

There is a lovely harmonic shift driven by the melody. The melody note in the C#7 measure is an F, which is a shift, because the digits had been centered around E major for so long (20+ notes) The next two measures (D -> A) brings relief of the shift to that F by going right back to E. The second half of this section has echos within itself. It also immediately shifts the tonal center. The "F" is back in the melody but in a completely different context than the previous C#7.

Take a moment to consider the odds of getting this much musical nuance and structure from rolling a 12 sided die 73 times (at this point in the song) and molding those notes to their very best

41

Bm      A#+      D      B      Em      B+      C#m7b5      D7

One of the few phrases that doesn't begin with a transition from the V. The tonal center is shifting again, now towards G major in the next section.

In the first half of this section (measures 41-44) the melody notes are ambiguous in harmonic movement. The melody notes spell out a D(add9) chord. I picked the chords to drive the first half of the melody by solving the second half of this section and mimicking that harmonic theme. The melody notes in measure 46 are a forced augmented chord. They work as a walkdown from Em. The C#m7b5 (C# G tritone) descends to the D7 (C F# tritone) which resolves to the G in the next section. The D7 is the most dissonant measure in the whole piece. To me, it's perfectly between forced and natural.

49

G Cm Dm G Em A7 F#7 B7

Coming to G from that D7(b9)(aug9) scenario, this section has 23 quarter notes in it, the most of any section so far. The melody notes drive the chords. The Cm was chosen over C major because in the key of G, the last 6th expressed was Eb. What you can do with common tones in this section is really amazing. It's hard to say when the tonal center shifts but it starts moving near the end.

57

Em C7 A#m F#7 C#m F#m A7 E7

Coming from B7, this section opens with Em to C7, then A#m to F#7, two changes with the same jump. i to flat VI

The melody notes are difficult to make flow here no matter how they are phrased and what harmony is added. (A# B C C# together as melody notes, in close proximity and awkward order)

The melody picks up again in the second half of the section. It's easy to see the chords in groups of two. There's a moderately stable tonal center, shifting every two measures.

The harmony is clearly driven by the melody notes.

C#m hits a flat 6 making F#m (i to iv). Tritone in melody makes A7 then tritone in melody makes E7 (I to V)

65

Gb7 B7 Gb Bbm Eb7 Abm E7 A

One of the few sections that doesn't come from a V in the previous section, this section carries on with the groupings of measures in two, driven by shifting harmony in the melody notes.

Gb to B7 is a I to IV

Gb to Bbm is I to iii

Eb to Abm is I to iv

E7 to A is I to IV

The repeating high Bb in the melody stands out in the phrase. Measures #65, 67, 69

73

Dbm A Ab7 Db7 Gm7 C5 F7 Bb5

This section is one of the stopping points in the piece. Anywhere a section ends with a Bb chord, it can resolve back to the very first phrase (in the key of Eb) and end.

The first half of this phrase is in Db ... Dbm - A - Ab7 - Db7 (i - bVI - V - I)

The second half is heading towards Eb ... Gm7 - C5 - F7 - Bb5 (iii - VI - II - V)

The Ab7 to Db7 is a I to IV

Gm7 to C5 is also a I to IV but a half step down from Ab7 Db7

C5 to F7 is I to IV, and so is F7 to Bb5

The melody notes in the C5 measure make the chord go from major to minor.

The melody notes in the Bb5 measure don't express the major, but the minor 3rd comes in.

81

Cm Fm Eb7 Ab F7 Bbm7(no5) Eb7 Ab

The next four sections are the most fascinating when examined together. The tonal center is clearly defined and moves slowly. It's currently headed for Db.

Notice the dissonant melody note in the 6th measure of the phrase

Cm to Fm is i to iv

Eb to Ab is I to IV

F7 to Bbm to Eb to Ab are all I to IV

The A natural in the melody (5th measure of the phrase) momentarily "raises" the scale.

89

DbMaj7 GbMaj7 Ebm Ab F7 Bb7 Ebm Ab7

Coming from Ab (the V) in to the key of Db ... I IV ii V III VI ii V

Notice the dissonant melody note in the 7th measure of the phrase

Once again the A natural in the melody (5th measure of the phrase) raises the scale, the same as in the previous phrase. This time though, the scale is raised twice in a row when the melody then goes to a D natural, making Bb major. III to VI instead of III to vi like the previous section. Look at the chords between this section and the previous.

From the 3rd measure on, the chords are the same but with some majors and minors changed.

97

8

Db Gb Bb7 Eb7(no5) Fm C7 Fm7 Bb7

This section starts in Db and shifts towards Eb. When the D natural comes in the melody of the third measure, the Db doesn't return.

The second half of the phrase (Fm to C7 to Fm7 to Bb7) has a nice walkdown in the harmony. F - E - Eb - D

Notice the ascending and descending arpeggios in the melody.

105

8

B7 Bb7 B7 Gb Bbm F B7( no5 ) Bb7(no5)

The tonal center was headed towards Eb, and was on a Bb7 at the end of the previous section, and then "boom" ... bVI This whole section is a big dramatic flat six to V turn around headed towards Eb.

Within the chords, there is a descending tone, some of which is chromatic Gb - F - Eb - Db - Db - C - B - Bb

The last two measures only need the root and dominant 7 to harmonize the melody notes, which can make nice big arpeggio if you have enough octaves. (raise the last 4 melody notes of the phrase up an octave) It's harder to link to the next phrase, octave wise (and the next phrase, and the next). This section is the culmination of the 4 phrases since the last ending point at 160 digits. It can arpeggiate up right back into the intro. This is probably the best point in the song to repeat and end (226 digits), and my personal favorite section in the piece.

113

B F# B F# F D7 Gm C7

From the B to Bb turnaround, back up and over-the-top to B again. B F# B F# ( I V I V )

The end of this phrase is an over-the-top arpeggio in the key of F ... F D7 Gm C7 ( I VI ii V )

121 F B $\flat$  G7omit3 C7 F7 C7 B7 E7

This section reminds me of Gershwin. The tonal center starts in F ( I IV ii V ) but shifts towards the end of the phrase.

129 Am Bm7 $\flat$ 5 E7 Am E7 F7 B $\flat$  E $\flat$

From an E7 in to Am ... ( i ii V i )

The melody drives the tonal center to shift abruptly at the end of this phrase.

Notice the melody intervals in measure 7. The melody notes at the end of the the next phrase are the same intervals, just transposed down a half step.

B $\flat$  - F - C - B $\flat$  in this phrase  
 A - E - B - A in the next phrase

137 Cm F7 B $\flat$ m G $\flat$  Bm7 Em7 A7 D7

Cm to F7 = i to IV  
 F7 to B $\flat$ m = I to iv  
 G $\flat$  to Bm7 = I to iv  
 Bm7 to Em7 = i to iv  
 Em7 to A7 = i to IV  
 A7 to D7 = I to IV

145

E $\flat$  B7 E B A $\flat$ 7 D $\flat$ m B $\flat$ 7 Fm

A strange cluster of notes. Different from the tonal centers around it. This is the only section I think might have a better cohesive flowing harmony than the chords I chose.

153

B $\flat$  G Cm B B $\flat$  F7 B $\flat$ 7 E7

It was all the double notes hovering around B $\flat$ 7 that hinted at this whole phrase as being a big "turn around" for another Eb resolve repeat/ending spot in the song.

1st half - Key of B $\flat$  ( I VI ii bII) ... 2nd half - Key of E $\flat$  ( V II V bII)

I'm not sure if "flat II" is the correct nomenclature for either of those scenarios.

The last measure is a tritone substitution for the V (B $\flat$ 7).

I love how the melody notes are sour in just the right places to allow the B and E7 to fall in measures 4 and 8 of the phrase.

161

E $\flat$  A $\flat$  Cm G7 A $\flat$  B $\flat$  Fm7 B $\flat$ 7 E $\flat$

From the 160th, 226th, or all the way up to the 330th digit, the song returns back to Eb and the beginning.

I only had to change one note to end the song.

The 18th digit of pi is a 5. I made it a 4 to end the song.

It makes a "44", which is a nice answer to the "66" right before it.

3 . 1 848 0 949 3B9 186 64 4